# J.K. SHAH CLASSES

## MATHEMATICS & STATISTICS

### SYJC TEST – 04 – SET 1

### DURATION – 1 $^1/_2$ HR                                    MARKS – 40

## SECTION – I

**Q1.**  **Attempt any THREE of the following**  (2 marks each)                                    **(6 marks )**

01.    the price P for demand D is given as :  $P = 183 + 120D - 3D^2$ . Find D for which the price is increasing

**SOLUTION**

₹

$$P = 183 + 120D - 3D^2$$

For price increasing

$$\frac{dP}{dD} > 0$$

$$120 - 6D > 0$$

$$120 > 6D$$

$$20 > D$$

$$D < 20 \qquad \textbf{ans : Price is increasing for D < 20}$$

*SOLUTION SET*

02.    Find the elasticity of demand , if the marginal revenue is 50 and price is 75

**SOLUTION**

$$Rm = R_A \left( 1 - \frac{1}{\eta} \right)$$

$$50 = 75 \left( 1 - \frac{1}{\eta} \right)$$

$$\frac{50}{75} = 1 - \frac{1}{\eta}$$

$$\frac{2}{3} = 1 - \frac{1}{\eta}$$

$$\frac{1}{\eta} = 1 - \frac{2}{3}$$

$$\frac{1}{\eta} = \frac{1}{3}$$

$$\eta = 3$$

03.   Find Marginal Propensity to Consume and Marginal Propensity to save if the expenditure $E_c$ of a person with income I is given as $E_c = (0.0003)I^2 + (0.075)I$ when $I = 1000$

**SOLUTION**

$$E_c = 0.0003I^2 + 0.075I$$

$$\text{MPC}\Big|_{I=1000} = \frac{dE_c}{dI}$$

$$= \frac{d\ 0.0003\ I^2 + 0.075\ I}{dI}$$

$$= 0.0006\ I + 0.075$$

$$= 0.0006(1000) + 0.075$$

$$= 0.6 + 0.075$$

$$= \mathbf{0.675}$$

$$\text{MPS}\Big|_{I=1000} = 1 - \text{MPC}\Big|_{I=1000}$$

$$= 1 - 0.675$$

$$= \mathbf{0.325}$$

**Q2.** **Attempt any TWO of the following** **(3 marks each)** **(6 marks )**

01. A firm wants to maximize its profit . The total cost function is C = 370Q + 550 and revenue R = 730Q − 3Q$^2$ . Find the output for which profit is maximum

**SOLUTION**

**STEP 1 :** Profit

$\pi$ = R − C

$\pi$ = 730Q − 3Q$^2$ − (370Q + 550)

$\pi$ = 730Q − 3Q$^2$ − 370Q − 550

$\pi$ = 360Q − 3Q$^2$ − 550

**STEP 2 :**

$\dfrac{d\pi}{dQ}$ = 360 − 6Q

$\dfrac{d^2\pi}{dQ^2}$ = −6

**STEP 3 :**

$\dfrac{d\pi}{dQ}$ = 0

360 − 6Q = 0

360 = 6Q

Q = 60

**STEP 4 :**

$\dfrac{d^2\pi}{dQ^2}\bigg|_{Q\ =\ 60}$ = −6 < 0

Profit is maximum at Q = 60

02.  For the demand law  $D = \dfrac{36}{p + 1}$

find elasticity of demand at p = 2 . State your conclusion

**SOLUTION**

**STEP 1 :**  $D = \dfrac{36}{p + 1}$

$$\dfrac{dD}{dp} = \dfrac{(p+1)\,\dfrac{d}{dp}\,36 - 36\,\dfrac{d}{dp}\,(p + 1)}{(p + 1)^2}$$

$$= \dfrac{(p + 1).0 - 36\,(1)}{(p + 1)^2}$$

$$= \dfrac{-36}{(p + 1)^2}$$

**STEP 2 :**  $\eta = \dfrac{-P}{D} . \dfrac{dD}{dp}$

$$= \dfrac{-p}{\dfrac{36}{p + 1}} . \dfrac{-36}{(p + 1)^2}$$

$$= \dfrac{p}{p + 1}$$

**STEP 3 :**  $\eta \Big|_{p = 2} = \dfrac{2}{2 + 1}$

$$= \dfrac{2}{3} < 1$$  Conclusion : Demand is relatively inelastic

03.  find values of x for which  $f(x) = 4x^3 - 12x^2 - 36x + 1$   is decreasing

**SOLUTION**

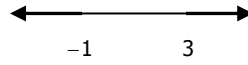For f(x) decreasing ,

$f'(x) < 0$

$12x^2 - 24x - 36 < 0$

$x^2 - 2x - 3 < 0$

$(x - 3)(x + 1) < 0$

**CASE 1 :**

    $x - 3 > 0$ & $x + 1 < 0$

    $x > 3$ & $x < -1$

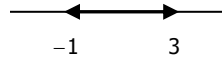                                                         −1        3

    NOT POSSIBLE SO DISCARD

**CASE 2 :**

    $x - 3 < 0$ & $x + 1 > 0$

    $x < 3$ & $x > -1$

                                                         −1        3

    $-1 < x < 3$

    $x \in (-1, 3)$

**$f$ is decreasing for $x \in (-1, 3)$**

**Q3.** **Attempt any TWO of the following** **(4 marks each)** **(8 marks )**

01. Examine the function $f(x) = x + \dfrac{25}{x}$ for maxima and minima

**STEP 1 :**

$$f(x) = x + \dfrac{25}{x}$$

**STEP 2 :**

$$f'(x) = 1 - \dfrac{25}{x^2} = \boxed{1 - 25x^{-2}}$$

$$f''(x) = 0 + 50x^{-3}$$

$$= \dfrac{50}{x^3}$$

**STEP 3 :**

$$f'(x) = 0$$

$$1 - \dfrac{25}{x^2} = 0$$

$$1 = \dfrac{25}{x^2}$$

$$x^2 = 25$$

$$x = \pm 5$$

Minimum value of $f$

$$= f(x) \Big|_{x = 5}$$

$$= 5 + \dfrac{25}{5}$$

$$= 10$$

**STEP 4 :**

$$f''(x)\Big|_{x = 5} = \dfrac{50}{5^3} > 0$$

$f$ is minimum at x = 5

Since $f$ is maximum at x = −5

Maximum value of $f$

$$= f(x) \Big|_{x = -5}$$

$$= -5 + \dfrac{25}{-5}$$

$$f''(x)\Big|_{x = -5} = \dfrac{50}{(-5)^3} < 0$$

$f$ is maximum at x = −5

$$= -5 - 5$$

$$= -10$$

**STEP 5 :**

Since $f$ is minimum at x = 5            cont.

02. The total cost function of a firm is $C = x^2 + 75x + 1600$ for output x. Find the output for which average cost is minimum. Also verify at this output $C_A = C_M$

**SOLUTION**

**STEP 1 : AVERAGE COST**

$C_A = \dfrac{C}{x}$

$= \dfrac{x^2 + 75x + 1600}{x}$

$= x + 75 + \dfrac{1600}{x}$

**STEP 2 :**

$\dfrac{dC_A}{dx} = 1 - \dfrac{1600}{x^2} = 1 - 1600x^{-2}$

$\dfrac{d^2C_A}{dx^2} = 0 + 3200x^{-3}$

$= \dfrac{3200}{x^3}.$

**STEP 3 :**

$\dfrac{dC_A}{dx} = 0$

$1 - \dfrac{1600}{x^2} = 0$

$1 = \dfrac{1600}{x^2}$

$x^2 = 1600$

$x = 40$     (output cannot be – ve)

**STEP 4 :**

$\left.\dfrac{d^2C_A}{dx^2}\right|_{x = 40} = \dfrac{3200}{40^3} > 0$

Average Cost is minimum at x = 40

**STEP 5 :**

$\left.C_A\right|_{x = 40} = 40 + 75 + \dfrac{1600}{40}$

$= 40 + 75 + 40$

$= 155$

**STEP 6 :**

$C_M = \dfrac{dC}{dx}$

$= 2x + 75$

$\left.C_M\right|_{x = 40} = 2(40) + 75$

$= 80 + 75$

$= 155$

Hence     $C_A = C_M$ at x = 40

Cont.

03. Demand function x , for a certain commodity is given as x = 200 − 4p , where p is the price  .

        Find  i) elasticity of demand as function of p

              ii) elasticity of demand when p = 10  . Interpret the results

              iii) the price p for which elasticity of demand is equal to one

**SOLUTION**

**STEP 1 :**    x = 200 − 4p

$$\frac{dx}{dp} = -4$$

**STEP 2 :** $\eta = \frac{-P}{D} \cdot \frac{dD}{dp}$

$$= \frac{-p}{x} \cdot \frac{dx}{dp}$$

IN THIS SUM DEMAND 'D' IS DENOTED AS 'X'

$$= \frac{-p}{200 - 4p} \cdot -4$$

$$= \frac{p}{50 - p}$$

**STEP 3 :** $\eta \Big|_{p = 10} = \frac{10}{50 - 10}$

$$= \frac{10}{40}$$

$$= \quad 0.25 \ < \ 1$$

Demand is relatively inelastic

**STEP 4 :** $\eta \Big|_{p = 30} = \frac{30}{50 - 30}$

$$= \frac{30}{20}$$

$$= \quad 1.5 \ > \ 1$$

Demand is relatively elastic

**STEP 2 :** $\eta = \frac{p}{50 - p}$

$$1 = \frac{p}{50 - p}$$

$$50 - p \ = \ p$$

$$p \ = \ 25$$

# SECTION - II

**Q4.** **Attempt any THREE of the following** (2 marks each)                    **(6 marks )**

01.   what is the sum due of ₹ 5,000 due 4 months hence at 12.5% p.a. SI

**SOLUTION**

F.V.  = P.W.+ INT ON P.W. FOR 4 MONTHS @12.5% p.a.

$$F.V. = 5000 + 5000 \times \frac{4}{12} \times \frac{12.5}{100}$$

$$F.V. = 5000 + 5000 \times \frac{1}{3} \times \frac{125}{1000}$$

F.V.  =   5000  +  208.33

F.V.  =   ₹ 5208.33


02.   Find the amount accumulated after 2 years if sum of ₹ 12,000 is invested at the end of every 6 months at 12% p.a. compounded half yearly   ( $1.06^4 = 1.2625$)

**SOLUTION :**

C = ₹ 12,000   ;   i = 6% = 0.06   ;   n = 4

$$A = C \left[ \frac{(1 + i)^n - 1}{i} \right]$$

$$= 12000 \left[ \frac{(1 + 0.06)^4 - 1}{0.06} \right]$$

$$= 12000 \left[ \frac{1.06^4 - 1}{0.06} \right]$$

$$= 12000 \left[ \frac{1.2625 - 1}{0.06} \right]$$

$$= 12000 \frac{0.2625}{0.06}$$

$$= 12000 \frac{26.25}{6}$$

$$= 2000 \times 26.25$$

$$= ₹ 52,500$$

03. a shop is valued at ₹ 2,40,000 for 75% of its value . If the rate of premium is 90 paise percent , find the premium paid by the owner of the shop . If the agent gets commission at 15% of the premium , find the agents commission

**Solution**

Property value = ₹ 2,40,000

Insured value = $\frac{75}{100}$ x 2,40,000

= ₹ 1,80,000

Rate of premium = 90 paise percent

= 0.90%

Premium = $\frac{0.9}{100}$ x 1,80,000

= $\frac{9}{1000}$ x 1,80,000

= ₹ 1,620

Agents commission

= $\frac{15}{100}$ x 1620

= ₹ 243

04. find the present value of an ordinary annuity of ₹ 18,000 p.a. for 3 years at 9% p.a. compounded annually ($1.09^{-3} = 0.7722$ )

**SOLUTION :**

C = ₹ 18,000 ; i = 9% = 0.09 ; n = 3

$P = C \left[ \frac{1 - (1 + i)^{-n}}{i} \right]$

= $18000 \left[ \frac{1 - (1 + 0.09)^{-3}}{0.09} \right]$

= $18000 \left[ \frac{1 - 1.09^{-3}}{0.09} \right]$

= $18000 \left[ \frac{1 - 0.7722}{0.09} \right]$

= $18000 \frac{0.2278}{0.09}$

= $18000 \frac{22.78}{9}$ = 2000 x 22.78 = ₹ 45,560

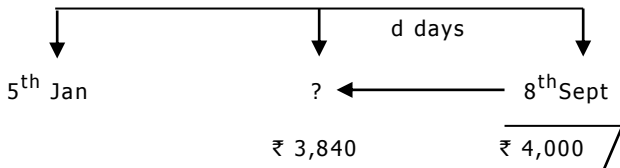01.   a bill of ₹ 4,000 drawn on 5th Jan 1998 for 8 months and was discounted for ₹ 3,840. Find the date on which the bill was discounted at 10% p.a.

**SOLUTION**

due 8 months   @ 10% p.a.

d days

5th Jan            ?  ←———— 8th Sept

₹ 3,840         ₹ 4,000

**STEP 1  :**

| | | |
|---|---|---|
| Date of drawing | = | 5 / 01 |
| Add period of bill | + | 8 months |
| Nominal due date | = | 5 / 09 |
| Add Grace days | + | 3 days |
| Legal due date | = | 8 / 09  ……. |

8th September

**STEP 2  :**

Let Unexpired period    = d days

**STEP 3  :**

B.D.  =  F.V. − C.V.

=  4,000 − 3,840

=  ₹ 160

**STEP 4  :**

B.D.  =  Interest on F.V. for 'd' days @ 10% p.a.

$$160 = \frac{4000 \times d \times 10}{365 \quad \quad 100}$$

$$d = \frac{160 \times 73}{80}$$

d  =  146 days

**STEP 5  :**

Dt. of Discount

=  8th Sept − 146 days

| Sept | Aug | Jul | Jun | May | Apr |
|---|---|---|---|---|---|
| = 8 | + 31 | + 31 | + 30 | + 31 | + 15 |

=  15th April

02. Mrs. Menon plans to save for her daughter's marriage . She wants to accumulate a sum of ₹ 4,00,000 at the end of 4 years . how much should she invest at the end of each year from now , if she can get interest compounded at 10% p.a. ( $1.1^4 = 1.4641$)

**SOLUTION :**

A = ₹ 4,00,000 ;  i = 10% = 0.1  ;  n = 4

$$A = C\left[\frac{(1 + i)^n - 1}{i}\right]$$

$$4,00,000 = C\left[\frac{(1 + 0.1)^4 - 1}{0.1}\right]$$

$$4,00,000 = C\left[\frac{(1.1)^4 - 1}{0.1}\right]$$

$$4,00,000 = C\left[\frac{1.4641 - 1}{0.1}\right]$$

$$4,00,000 = C\left(\frac{0.4641}{0.1}\right)$$

$$4,00,000 = C\left(\frac{4.641}{1}\right)$$

$$C = \frac{4,00,000}{4.641}$$

$$= ₹ 86,200$$

LOG CALC

5.6021
− 0.6666
———————
AL 4.9355

86200

03. A person aged 35 years takes a policy for ₹ 1 lakh for a period of 20 years . The rate of premium is ₹ 76 and the average bonus rate is ₹ 7 per thousand per annum . If he dies after paying 10 annual premiums , find the total premium paid and what will the nominee get .

**SOLUTION**

Policy value  =  ₹ 1,00,000

STEP − 1 : TOTAL PREMIUM PAID

Rate of premium

$$= ₹ \text{ 76 per thousand per annum}$$

Premium     $= \dfrac{76}{1000} \times 1,00,000$

$$= ₹ \text{ 7,600 p.a}$$

Premium paid

In 10 years    = 7600 x 10

$$= ₹ \text{ 76,000}$$

STEP − 2 : TOTAL BONUS ACCRUED

Rate of Bonus

$$= ₹ \text{ 7 per thousand per annum}$$

Bonus     $= \dfrac{7}{1000} \times 1,00,000$

$$= ₹ \text{ 700 p.a}$$

Bonus accrued

In 10 years    = 700 x 10

$$= ₹ \text{ 7,000}$$

STEP − 3 : NOMINEE GETS

Nominee gets   = sum assured  + Bonus accrued

$$= 1,00.000 + 7,000$$

$$= ₹ \text{ 1,07,000}$$

01. Property valued at ₹ 7 lakh is insured to the extent of ₹ 5,60,000 at 5/8 % less 20%. How much loss does the owner bear including premium if the property is damaged to the extent of 40% of its value

**Solution**

Property value = ₹ 7,00,000

Insured value = ₹ 5,60,000

Rate of premium = 5/8 % less 20%.

Premium = $\dfrac{5}{800}$ x 5,60,000

= ₹ 3,500

less 20% disc − 700
_____

Net Premium = ₹ 2,800

Loss = $\dfrac{40}{100}$ x 7,00,000

= ₹ 2,80,000

Claim = $\dfrac{\text{insured val.}}{\text{Property val.}}$ x loss

= $\dfrac{5,60,000}{7,00,000}$ x 2,80,000

= ₹ 2,24,000

| Loss | = | 2,80,000 |
|------|---|----------|
| Less claim | − | 2,24,000 |
| Net loss | = | 56,000 |
| Add premium | + | 2,800 |
| Net loss Incl. premium | = ₹ | 58,800 |

02. Find the true discount , banker's discount and banker's gain on a bill of ₹ 36,600 due 4 months hence discounted at 5% p.a

**SOLUTION**

**STEP 1 :**

FV = PW + Int on PW for 4 months @ 5% p.a.

$$36600 = PW + PW \times \frac{4}{12} \times \frac{5}{100}$$

$$36600 = PW + \frac{PW}{60}$$

$$36600 = \frac{61 \, PW}{60}$$

$$PW = \frac{36600 \times 60}{61}$$

$$= ₹ 36,000$$

**STEP 2 :**

TD = Int on PW for 4 months @ 5% p.a.

$$= 36000 \times \frac{4}{12} \times \frac{5}{100}$$

$$= ₹ 600$$

**STEP 3 :**

BD = Int on FV for 4 months @ 5% p.a.

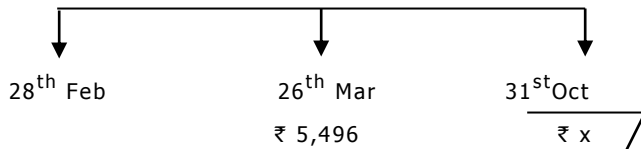$$= 36600 \times \frac{4}{12} \times \frac{5}{100}$$

$$= ₹ 610$$

**STEP 4 :**

BG = BD − TD

= 610 − 600

= ₹ 10

03. A bill of a certain amount drawn on 28th February 2007 for 8 months was cashed on 26th March 2007 for ₹ 5,496 at 14% p.a . Find the face value of the bill

**SOLUTION**    due 8 months    @ 14 % p.a.

28th Feb          26th Mar          31st Oct
                  ₹ 5,496           ₹ x

**STEP 1  :**

Date of drawing       =     28 / 02

Add period of bill       +     8 months

Nominal due date      =     28 / 10

Add Grace days         +     3 days

Legal due date         =     31 / 10      …….

                        31st Oct

**STEP 2  :**

Unexpired period

=  26th Mar  −  31th Aug

MAR   APR   MAY   JUN   JUL   AUG   SEP   OCT

=5  + 30 + 31 + 30 + 31 + 31 + 30 + 31

=  219 days

**STEP 3  :**

B.D.  =  F.V.  −  C.V.

      =  x  −  5496

**STEP 4  :**

B.D.  =  Interest on F.V. for 158 days @ 14% p.a.

$$x - 5496 = \text{'x'} \times \frac{\overset{3}{\cancel{219}}}{\underset{5}{\cancel{365}}} \times \frac{\overset{7}{\cancel{14}}}{\underset{50}{\cancel{100}}}$$

$$x - 43500 = \frac{21x}{250}$$

$$x - \frac{21x}{250} = 5496$$

$$\frac{229x}{250} = 5496$$

$$x = \frac{\overset{24}{\cancel{5496}} \times 250}{\cancel{229}}$$

x      =  ₹ 6000